Error suppression in continuous-time quantum computing

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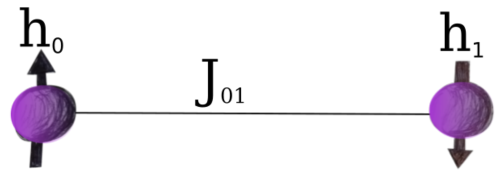
1. Summary

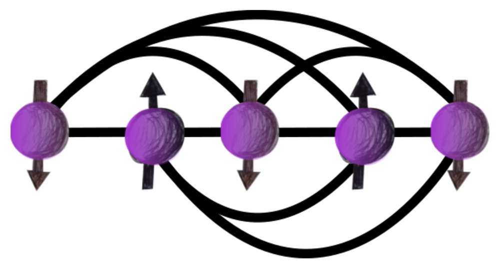
- Quantum Computers may be faster and/or more efficient at solving **optimisation problems** than classical computers. E.g. Max2Sat, MIS.
- Continuous-time quantum computing; **Quantum walks, Adiabatic Quantum Computing (AQC) and Quantum Annealing**, is an ideal setting for optimisation problems.
- However, there is limited **error correction** capabilities of continuous-time quantum computing.
- We connect **3 copies** of an **Ising spin glass** in a loop with **anti-ferromagnetic** couplings to improve **robustness** to error.

2. Transverse Ising model

- Classical optimization problems can be encoded onto **Ising models.** Answer is the **ground state.**



- We use Sherrington-Kirkpatrick **Ising spin glasses:**



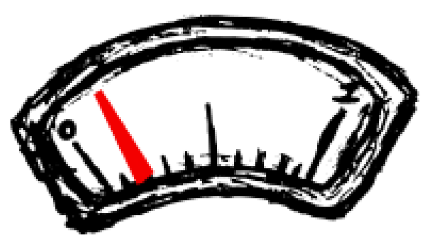
- Uniformly hard, NP-hard.
- Fields, h_i, couplings, J_{ij}. Drawn randomly from uniform range [-1, 1].

$$\hat{H}_P = \sum_{i=0}^{n-1} h_i \hat{Z}_i + \sum_{i \neq j=0}^{n-1} J_{ij} \hat{Z}_i \hat{Z}_j$$

3. Continuous-time Quantum computing

- Solve using **continuous-time** technique.
- Initial state: $|\psi(0)\rangle = \frac{1}{\sqrt{N}} \sum_{j=0}^{n-1} |j\rangle$
- Drive into **ground state** of $\hat{H}_P$:

$\hat{H} = A(t)\hat{H}_I + B(t)\hat{H}_P$



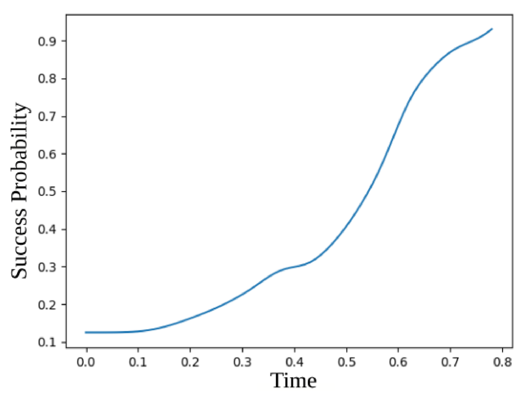
Using:

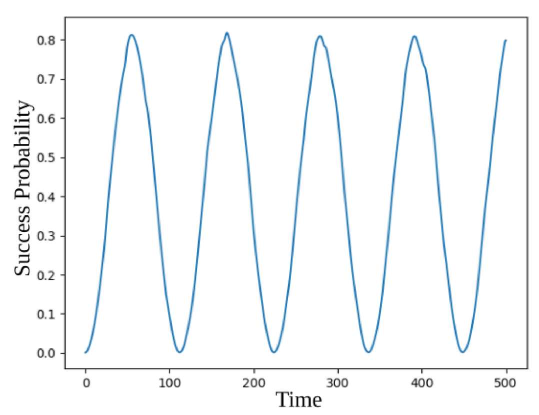
- Adiabatic Quantum Computing (AQC)

$\hat{H}_I = n\hat{1} - \sum_{j=0}^{n-1} \hat{X}_j$

- Quantum Walk

$\hat{H}_I = \gamma(n\hat{1} - \sum_{j=0}^{n-1} \hat{X}_j)$

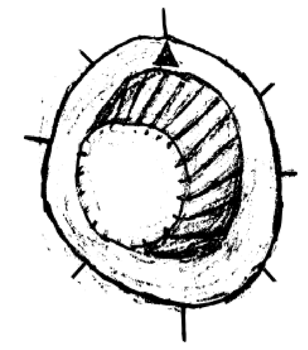


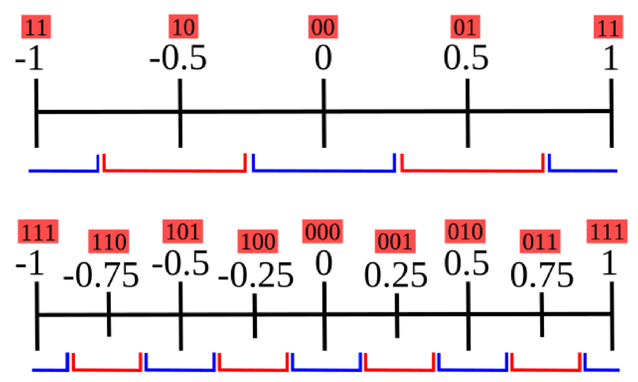


4. Error model

- We model limitations in the **resolution** of h's and J's.
- Define number of allowed values, in interval $[-1, 1]$:

$n_v = 2^p + 1$, where p is **precision**.

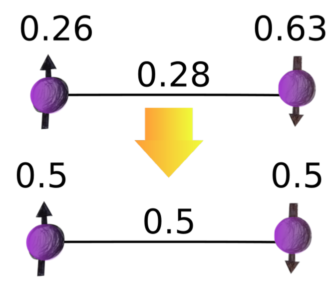




← allowed values for $p = 2$ and $p = 3$.

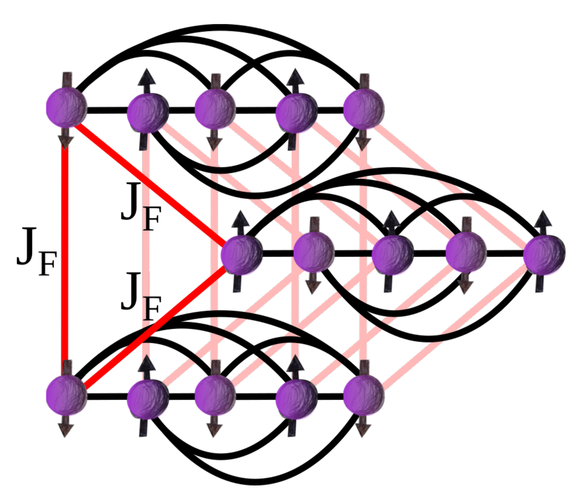
- Uncertainties in red and blue.

- When $p = 2$, an Ising model with true precision becomes →
- If this change is **too large** a qubit could flip → **incorrect** ground state!



5. Error suppression method

- We aim to find the **correct** ground state with **lower** precision.
- We connect 3 copies **anti-ferromagnetically** in a loop, causing **frustration**.



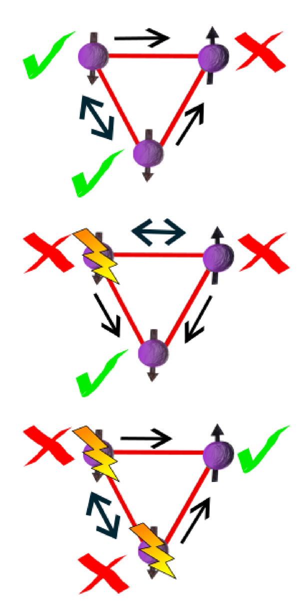
The Hamiltonian for this is,

$$\hat{H} = \sum_{i=0}^{n-1} \sum_{k=0}^{m-1} (h_i + \epsilon^h_{i,k}) \hat{Z}_{i,k} + \sum_{i \neq j=0}^{n-1} \sum_{k=0}^{m-1} (J_{ij} + \epsilon^J_{i,j,k}) \hat{Z}_{i,k} \hat{Z}_{j,k}$$

$$- \sum_{i=0}^{n-1} \sum_{k \neq l=0}^{m-1} F Z_{i,k} Z_{i,l}$$

6. Frustration

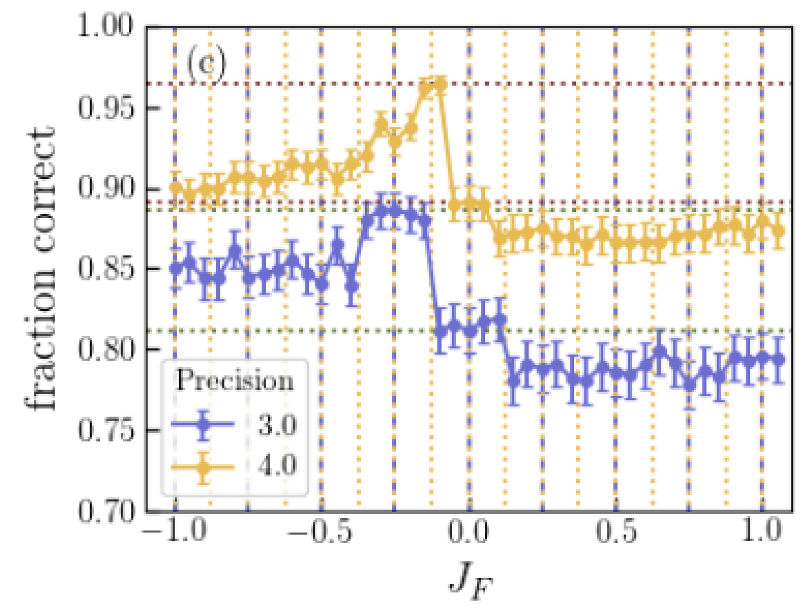
- Frustration inhibits: **error propagation**.
- **No errors:**
One copy is forced to be incorrect.
- **One error:**
One copy is forced to be correct.
- **Two errors:**
One copy is forced to be correct.



- We only need **one or more** copies to be correct.

7. Optimal link strength

- Need to find **optimal strength** for couplings J_F linking copies.



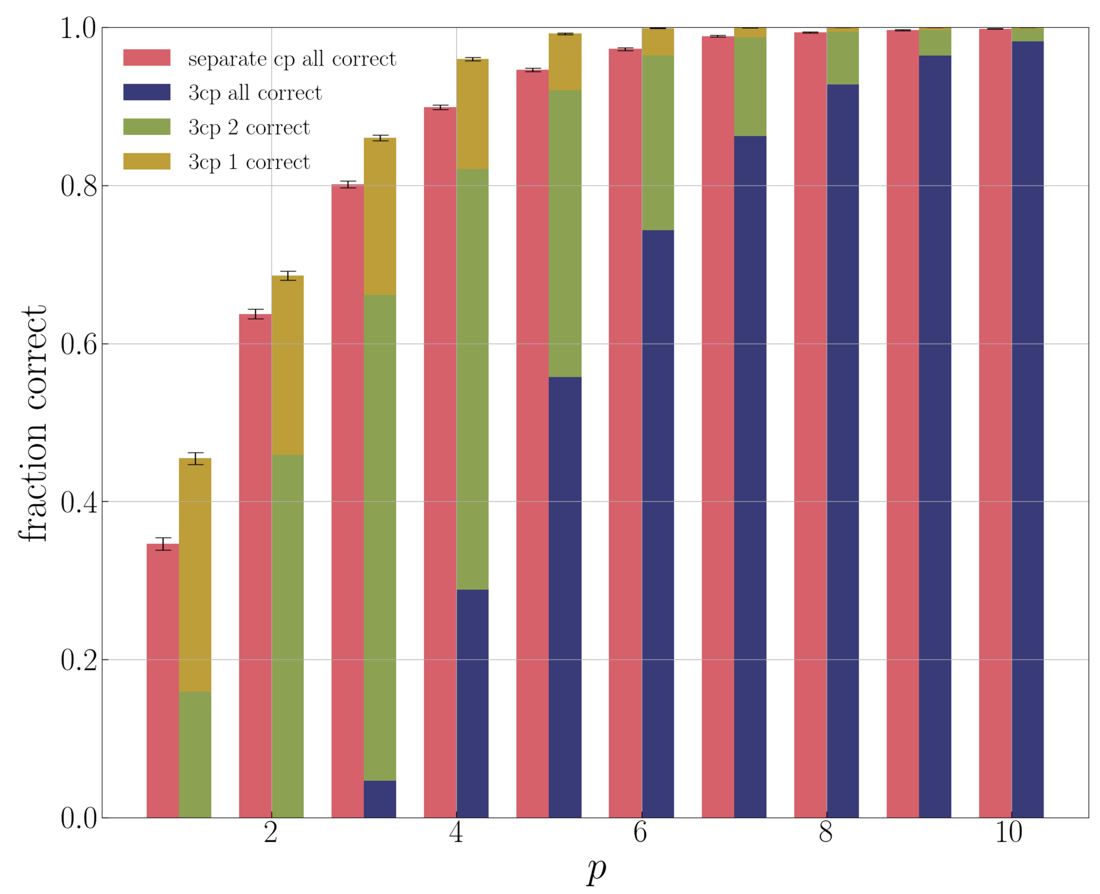
- 1000 5 qubit Ising spin glasses
- Varied J_F between -1 and 1.
- **Fraction correct:** fraction of times we find the correct ground state.
- At $p = 3$ and $p = 4$.
- Optimal setting: J_F to **minimum** (negative) value allowed by precision.

8. Results

- Subjected 10000 5 qubit spin glasses to precision p.

Measured **fraction correct** by:

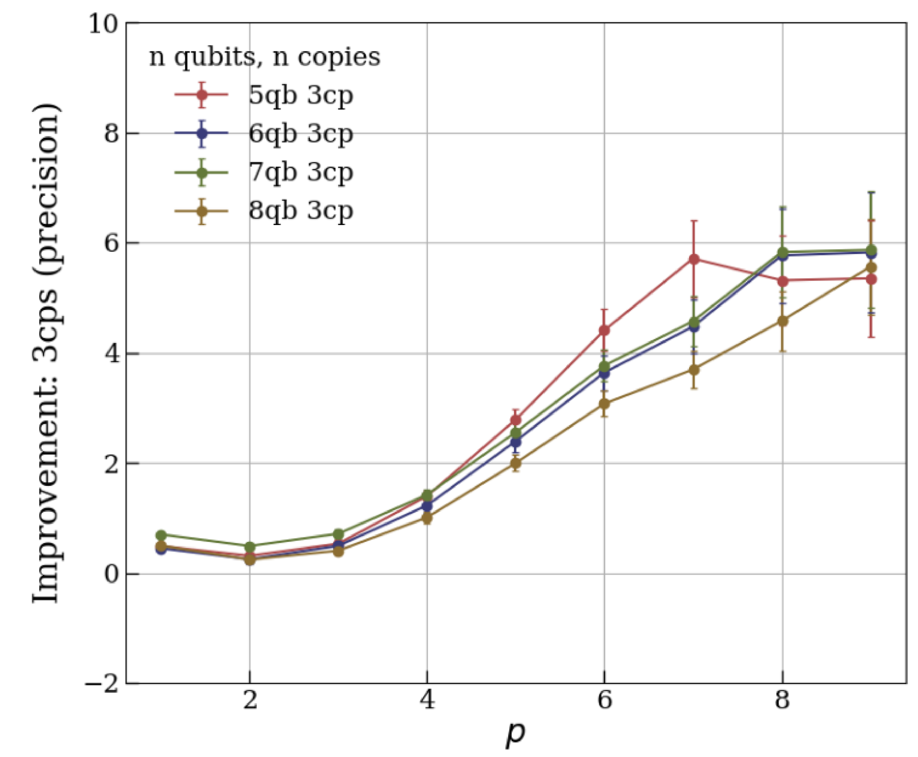
- Finding **ground state** of each **single copy** (equiv. to 3 separate) using classical branch and bound technique. → LHS bars
- Did the same for **3 copies** connnected anti-ferromagnetically (loop). → RHS bars: 3 copies correct (blue), 2 copies correct (green), 1 copy correct (yellow).



- 3 connected copies **outperforms** single copies.
- Trend continues for **larger** spin glasses. (6, 7, 8, 9 qubit spin glasses were tested).

9. Precision improvements

- Measured the difference (**improvement**) in precision between 1 and 3 copies of 5, 6, 7, 8 qubit spin glasses.



- Improvement in precision (in bits) **increases** with increasing precision of the spin glass.
- The improvement trend remains similar as the size of spin glass gets **larger**.

10. Conclusion

- Connecting **3 copies** of Ising **spin glasses** in a **loop** with **anti-ferromagnetic** links, set to **minimum** allowed precision, is **more robust** to lack of precision compared to three disconnected copies.
- Trend continues for **larger** (5, 6, 7, 8 qubit) spin glasses.
- Connecting copies technique, could be used to **increase effective precision** of computation.
→ This enables bypassing of limitations in hardware precision which arise as qubit number increases.

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